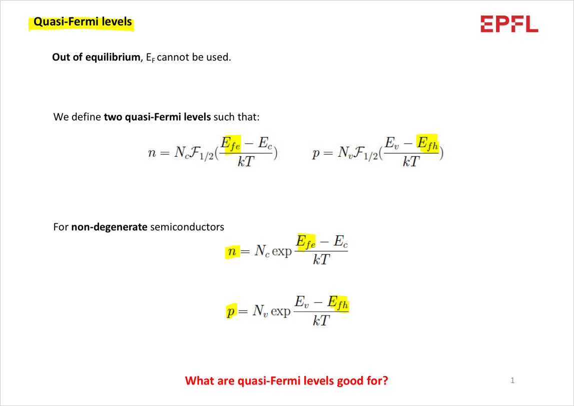
Course 10

Monday, 13 October 2025 15:10

EE557 Cours 10 - exerci...



Quasi-Fermi levels

Out of equilibrium, E_F cannot be used.

We define **two quasi-Fermi levels** such that:

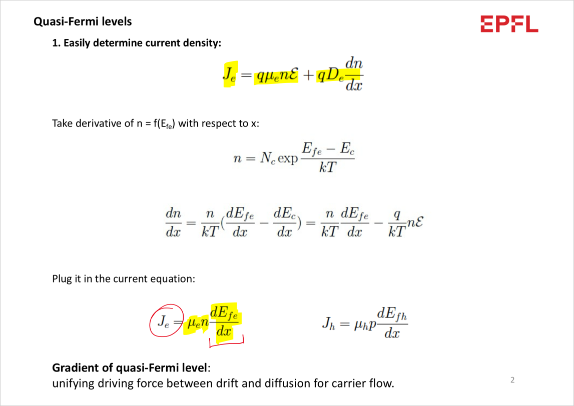
$$n = N_c \mathcal{F}_{1/2}(\frac{E_{fe} - E_c}{kT}) \qquad p = N_v \mathcal{F}_{1/2}(\frac{E_v - E_{fh}}{kT})$$

For **non-degenerate** semiconductors

$$n = N_c \exp \frac{E_{fe} - E_c}{kT}$$

$$p = N_v \exp \frac{E_v - E_{fh}}{kT}$$

What are quasi-Fermi levels good for?



Quasi-Fermi levels

1. Easily determine current density:

$$J_e = q\mu_e n\mathcal{E} + qD_e \frac{dn}{dx}$$

Take derivative of $n = f(E_{fe})$ with respect to x:

$$n = N_c \exp \frac{E_{fe} - E_c}{kT}$$

$$\frac{dn}{dx} = \frac{n}{kT}(\frac{dE_{fe}}{dx} - \frac{dE_c}{dx}) = \frac{n}{kT}\frac{dE_{fe}}{dx} - \frac{q}{kT}n\mathcal{E}$$

Plug it in the current equation:

$$J_e = \mu_e n \frac{dE_{fe}}{dx} \qquad J_h = \mu_h p \frac{dE_{fh}}{dx}$$

Gradient of quasi-Fermi level:
unifying driving force between drift and diffusion for carrier flow.

Quasi-Fermi levels

Physical meaning of ∇E_f

For electrons,

$$J_e = \mu_e n \frac{dE_{fe}}{dx} = -qnv_e$$

$$\frac{dE_{fe}}{dx} = -\frac{q}{\mu_e} v_e$$

∇E_{fe} is linearly proportional to electron velocity!

Similarly for holes:

$$\frac{dE_{fh}}{dx} = \frac{q}{\mu_h} v_h$$

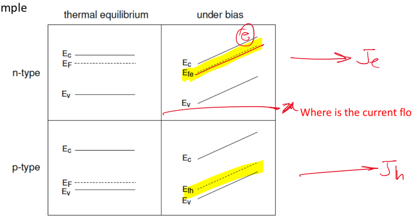
Quasi-Fermi levels

2. Visualize currents

$$J_e = \mu_e n \frac{dE_{fe}}{dx}$$

- $\nabla E_{fe} = 0 \;\Rightarrow\; J_e = 0$
- $\nabla E_{fe} \neq 0 \;\Rightarrow\; J_e \neq 0$
- if n high, ∇E_{fe} small to maintain a certain current level
- if n low, ∇E_{fe} large to maintain a certain current level

Example



Where is the current flowing?

Quasi-Fermi levels

3. Visualize carrier concentrations and net recombination

$$np = n_i^2 \exp \frac{E_{fe} - E_{fh}}{kT}$$

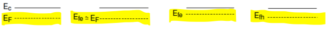
- if $R > G \;\rightarrow\; U > 0$, net recombination prevails
- if $R < G \;\rightarrow\; U < 0$, net generation prevails
- if $R = G \;\rightarrow\; U = 0$, thermal equilibrium

- If $E_{fe} > E_{fh} \Rightarrow np > n_i^2 \Rightarrow U > 0$
- If $E_{fe} < E_{fh} \Rightarrow np < n_i^2 \Rightarrow U < 0$
- If $E_{fe} = E_{fh} \Rightarrow np = n_i^2 \Rightarrow U = 0$ (carrier conc's in TE)

Examples (same semiconductor):

$p', n' \sim n_0$

$P_0 < n', p' < n_0$



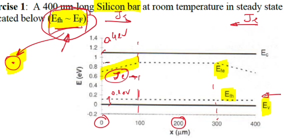
Efh
Ev
Efh
Efe

Thermal equilibrium | Low-injection | high-injection net recombination prevails | Extraction: net generation prevails

L.L.I

Exercises - list 2

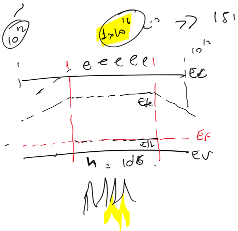
Exercise 1: A 400 um-long Silicon bar at room temperature in steady state has the band diagram indicated below ($E_{fh} \sim E_F$):



p-type

$p = N_v \cdot \exp\left(\frac{E_F - E_v}{KT}\right)$

$n = N_C \cdot \exp\left(\frac{E_c - E_F}{KT}\right)$ $\rightarrow E_{fe}$



a. Circle all the terms that apply to the situation:

- thermal equilibrium / out of equilibrium
- uniformly doped / non-uniformly doped
- n-type / p-type
- low level injection / high level injection/extraction
- intrinsic / extrinsic

b. Estimate the doping concentration. What is the amount of injected carriers (n') at 200um (middle) and 0 (origin) of the sample, compared to the equilibrium concentrations?

c. Estimate the electron current density at x = 0.

d. Estimate the gradient of electron concentration at x = 0.

e. Can you give an example of a situation that produces such band diagram?

b) $N_a \simeq P_o = N_v \cdot \exp\left(\frac{\overset{= E_{fh}}{E_F - E_v}}{KT}\right) = 6.6 \times 10^{17}\ cm^{-3} \Rightarrow \boxed{N_A = 6.6 \times 10^{17}\ cm^{-3}}$

$= 0.1\ eV$

$N_v = 3.1 \times 10^{19}$, $KT = 0.026$

$n_o = \frac{n_i^2}{P_o} = 151\ cm^{-3}$

at 200 µm : $E_c - E_{fe} = 0.2\ eV$

$n = N_c \exp\left(\frac{E_c - E_{fe}}{kT}\right) = 1.3 \times 10^{16}\ cm^{-3}$

$N_c = 2.9 \times 10^{19}$, $E_c - E_{fe} = 0.2$, $kT = 0.026$

$n' = n - n_o = 1.3 \times 10^{16} - 151 = 1.3 \times 10^{16}\ cm^{-3}$

at origin : $E_c - E_{fe} = 0.4\ eV \Rightarrow n = 6 \times 10^{12}\ cm^{-3}$

$n' = n - n_o = 6 \times 10^{12} - 151 = 6 \times 10^{12}\ cm^{-3}$

c) $J_e\Big|_{x=0} = n(x=0) \cdot \mu_e \cdot \frac{dE_{fn}}{dx}\Big|_{x=0}$

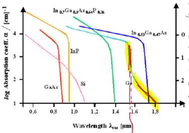
$n(0) = 6 \times 10^{12}\ cm^{-3}$

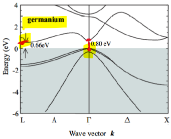
$\Rightarrow \boxed{J_e = 7.7\ mA/cm^2}$

$\mu_e = 400\ cm^2/V.s$

$\frac{dE_{fn}}{dx}\Big|_{x=0} = \frac{0.9 - 0.7}{100 - 0} = 20\ eV/cm$

Exercise 6. Consider the relationship between absorption coefficient and incident wavelength for germanium shown below (taken from the lecture slides, Course 7 page 20). Why do you think there is a sudden increase in absorption for wavelengths less than 1.55 µm? (please refer to the diagrams in appendix if needed)



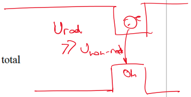


1.550 µm

$$\frac{1240}{1550} = 0.8eV$$

Exercise 8. The main function of an LED is to convert electrical energy into light in the visible part of the spectrum for display and illumination purposes. For a given input power, the radiative recombination processes are in direct competition with the nonradiative ones. Thus, the internal quantum efficiency (IQE) η_{iqe} is defined as:

$$\eta_{\text{iqe}} = \frac{\text{number of photons emitted internally}}{\text{number of carriers passing junction}}$$



It can be related to the fraction of the injected carriers that combine radiatively to the total recombination rate,

$$\eta_{\text{iqe}} = \frac{U_{\text{rad}}}{U_{\text{rad}} + U_{\text{nonrad}}} [1]$$

a. Under the assumption of low-level Injection (LLI) approximation, derive the expression of η_{iqe} in terms of lifetime for radiative, non-radiative (auger and trap-assisted) processes. Using these expressions, calculate the recombination lifetime of each mechanism for an n-type Si under quasi-neutral low level injection (LLI) with N_D=2x10^{17}cm^{-3} at room temperature. This sample contains a density of traps of N_t = 10^{16}cm^{-3} and the electron and hole capture rates are c_e=c_h=10^{-11}cm^{-3}.

---

b. Calculate the internal quantum efficiency for the case above.

c. Assuming a non-radiative recombination time of 5 ns for GaAs, r_{rad}=7×10^{-10}cm^{3}s^{-1} and heterostructure doping level of N_D=2×10^{17}cm^{-3}, find the internal quantum efficiency of the GaAs emitter.

d. Explain the reasons for the different IQE values between Si and GaAs.

e. What is the wavelength emitted by each of these materials (Si and GaAs). If you replace GaAs by GaP, what change in the wavelength is expected? Is this a suitable materials for light emission.

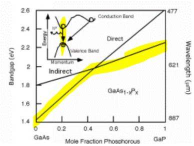
$$\lambda = \frac{1240}{BG.}$$

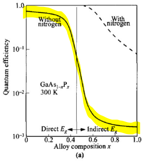
from the band gap:

Si : 1107 nm (IR)

GaAs : 689 nm (red)

GaP : 564 nm (green)





ⓐ η_{iqe} ?

$$U_{rad} = r_{rad}(np - n_0p_0) \overset{\Uparrow}{\underset{L.L.I}{\simeq}} r_{rad} \cdot n' \cdot (p_0 + n_0)$$

1) $U_{rad} = \dfrac{n'}{\tau_{rad}} \Rightarrow \tau_{rad} = \dfrac{1}{r_{rad}(n_0+p_0)} \overset{n\text{-type}}{\underset{\Downarrow}{=}} \dfrac{1}{r_{rad} \cdot n_0} = \dfrac{1}{400}\ s$

$r_{rad} = 2\times10^{-15}\ cm^{3}/s$, $n_0 = 2\times10^{17}\ cm^{-3}$

2) $\tau_{Aug} = \dfrac{1}{(r_{eeh} \cdot n_0 + r_{ehh} p_0)\cdot(n_0+p_0)} \overset{n\text{-type}}{\underset{\Downarrow}{=}} \dfrac{1}{r_{eeh} \cdot n_0^2} = \dfrac{1}{7200}\ s$

$r_{eeh} = 1.8\times10^{-31}\ cm^{6}/s$

3) $\tau_{tr} = \dfrac{\tau_{h0} n_0 + \tau_{e0}}{n_0 + p_0} \overset{n\text{-type}}{\underset{\Downarrow}{\Longrightarrow}} \tau_{tr} = \tau_{h0} = \dfrac{1}{N_t \cdot c_h} = \dfrac{1}{10000}\ s$

$N_t = 10^{16}\ cm^{-3}$, $c_h = 10^{-11}\ cm^{3}/s$

$$\eta_{iqe} = \frac{U_{rad}}{U_{rad} + U_{non\text{-}rad}} = \frac{400}{400 + 7200 + 10000} = 2.3\%$$

GaAs:

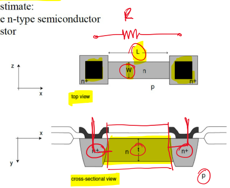
$\tau_{non\text{-}rad} = 5ns$

$\tau_{rad} = \dfrac{1}{2\times10^{17} \times 7\times10^{-10}} =$

$$\eta_{ife} = \frac{U_{rad}}{U_{rad} + U_{non\text{-}rad}} = 41\%$$

Exercise 9. Consider an integrated resistor composed of an n-type Si layer fabricated on a p-type Si. The doping level is $N_D = 10^{19} cm^{-3}$. The dimensions of its active region are L = 5um W = 2um and t = 0.1um. At RT, estimate:

1. Sheet resistance of the n-type semiconductor
2. Resistance of the resistor



$$\sigma = q \cdot n \cdot \mu_e$$

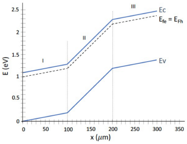
$$\rho = \frac{1}{\sigma} = \frac{1}{q \cdot n \cdot \mu_e \cong 100\ cm^2/Vs} \quad [\Omega . cm]$$

$$\underset{N_D = 10^{19}\ cm^{-3}}{n}$$

$$R_s\ [\Omega/\square] = \rho/t = \frac{5 \times 10^{-3}}{1 \times 10^{-5}} = 500\ \Omega/sq.$$

$$R = R_s \cdot \frac{L}{W} = 500 \cdot \frac{5\mu m}{2\mu m} = \boxed{1250\ \Omega}$$

Exercise 4. A Silicon device at room temperature in steady state has the band diagram shown below:



a. Select the terms that apply to the situation. Briefly justify your choices.

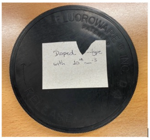
- thermal equilibrium / out of equilibrium
- which prevails: Recombination / Generation / none
- uniformly doped / non-uniformly doped
- n-type / p-type
- degenerate / non-degenerate
- low level injection / high level injection /extraction
- intrinsic / extrinsic

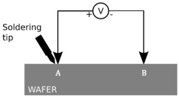
b. Calculate and plot the electric field, electrostatic potential (assume the reference 0V to be at the origin), electron concentration and hole concentration as a function of *x*, across the three regions. (Assume that the distance of the E_c-E_{fe} is the same throughout *x*)

c. Calculate and plot the total current density as a function of *x* across the three regions (pay attention to the units).

d. Describe in enough details an example of a situation that produces such band diagram. Explain in details and quantify your answer.

Exercise 3: You found a silicon wafer in the lab with a note: "Uniformely doped XX-type with $10^{16}cm^{-3}$". You need to use this wafer but you do not know whether it is p- or n-doped. With a hot soldering iron and a voltmeter (only measure voltages) that you found in the lab, can you figure out which doping type this wafer has?





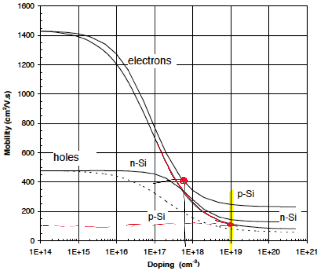
1. Before using the hot soldering tip, what voltage V do you expect to measure between A and B? which process dominates: recombination, generation or none?

After touching the hot soldering tip as in the figure above:

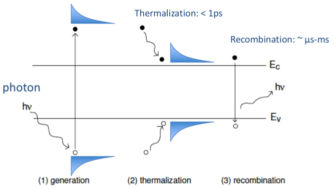
2. Is the wafer in thermal equilibrium?

3. What is the hot soldering tip doing to the majority and minority carriers (Assume that it is hot enough for low level injection)? Sketch the band diagram and position of the quasi-fermi levels in the case of n-type and p-type. Indicate the position of the hot soldering A and the position B.

4. Using the multimeter as indicated in the above figure, you measured a negative voltage ($V_B > V_A$), what can you conclude about the doping type? At the point A, which process dominates: recombination, generation or none? And at point B, far away from A?



Quasi-Fermi levels



Thermalization times are much shorter than recombination times